

**Inverse Dynamical Population Synthesis and Star Formation  
K1**

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**Abstract.** Recent observations of pre-main sequence stars suggest that all stars may form in multiple systems. However, in the Galactic field only about 50 per cent of all systems are binary stars. We investigate the hypothesis that stars form in aggregates of binary systems and that the dynamical evolution of these aggregates leads to the observed properties of binary stars in the Galactic field. A thorough analysis of star count data implies that the initial stellar mass function rises monotonically with decreasing mass and that it can be approximated by three power law segments. Together with our assumption that the birth mass-ratio distribution is not correlated this leads to a contradiction with the distribution of secondary masses in Galactic field binaries with G-dwarf primaries which have too few low-mass companions. For the inverse dynamical population synthesis we assume that the initial distribution of periods is flat in  $\log_{10}P$ , where  $P$  is the orbital period in days, and  $3 \leq \log_{10}P \leq 7.5$ . This is consistent with pre-main sequence data. We distribute  $N_{\text{bin}} = 200$  binaries in aggregates with half mass radii  $0.077 \leq R_{0.5} \leq 2.53$  pc, corresponding to the range from tightly clustered to isolated star formation, and follow the subsequent evolution of the stellar systems by direct N-body integration. We find that hardening and softening of binary systems do not significantly increase the number of orbits with  $\log_{10}P < 3$  and  $\log_{10}P > 7.5$ , respectively. After the cluster with  $R_{0.5} \approx 0.8$  pc disintegrates we obtain a population which consists of about 60 per cent binary systems with a period distribution for  $\log_{10}P > 4$  as is observed and in which the G-dwarf binaries have a mass ratio distribution which agrees with the observed distribution. This result indicates that the majority of Galactic field stars may originate from a clustered star formation mode, characterised by the *dominant mode cluster* which has initially  $(N_{\text{bin}}, R_{0.5}) \approx (200, 0.8$  pc). We invert the orbit depletion function and obtain an approximation to the initial binary star period distribution for star formation in the dominant mode cluster. Comparison with the measured distribution of orbits for pre-main sequence stars formed in the distributed mode of star formation suggests that the initial distribution of binary star orbits may not depend on the star formation environment. If a different stellar mass function to the one we adopt is assumed then inverse dynamical population synthesis cannot solve for an aggregate in which the initial binary star population evolves to the observed population in the Galactic field. This implies that the Galactic field stellar mass function may be related to the stellar density at birth in the most common, or dominant, mode of star formation.

**Keywords:** stars: low mass, mass function – binary stars: orbital parameters, evolution – stellar clusters: Galactic, embedded – star formation: distributed, clustered, dominant mode

## 1 INTRODUCTION

Although the formation of a single star appears to be understood fairly well a stimulating debate continues as to the formation mechanism of multiple star systems (see e.g. Bodenheimer et al. 1993). This has become a very important issue because during the recent two years observations have shown that the orbital period distribution and frequency of occurrence of pre-main sequence binary stars differs from that observed on the main sequence suggesting that virtually all pre-main sequence stars may be in binary systems.

Abt & Levy (1976, see also Abt 1987) pioneered the detailed study of statistical properties of orbital parameters of binary systems with a solar-type primary. Many of their findings are verified and improved by the long-term CORAVEL spectroscopic survey of nearby solar-type stars published by Duquennoy & Mayor (1991). Combined with data on visual binaries and common proper motion systems this survey has established the following four points of particular interest for the present study: 1) the proportion,  $f_G$ , of binary systems with a G dwarf primary is about 50–60 per cent, 2) the distribution of orbital periods,  $P$ , is approximated rather well by a Gaussian distribution in  $\log_{10}P$  (in what follows we measure  $P$  in days) with mean  $\log_{10}P = 4.8$ , 3) the eccentricity distribution for systems with orbital period  $\log_{10}P > 3$  approximately follows the thermal distribution but is bell shaped for smaller periods, and 4) the mass ratio distribution does not follow the expected distribution if the secondaries were distributed according to the field-star mass function: low-mass secondaries are underrepresented (Kroupa & Tout 1992). Systems with periods  $\log_{10}P > 3$  have a mass ratio distribution that increases with decreasing mass ratio but flattens with a possible decline below a mass ratio of approximately 0.25. For smaller periods the mass ratio distribution is flat or might even decrease with decreasing mass ratio (Mazeh et al. 1992). Similar results appear to hold for K-dwarf stars (Mayor et al. 1992) and M dwarfs (Fischer & Marcy 1992).

Occultation observations of pre-main sequence stars in the Taurus star-forming region supplemented by infrared imaging show that the proportion of binaries among pre-main sequence stars with separations between about 2 AU and 1400 AU is approximately 1.5 times larger than on the main sequence (Simon 1992). A CCD imaging survey of visual pre-main sequence binaries in southern dark clouds, all at a distance of about 150 pc, also suggests that the proportion of binaries among pre-main sequence stars is about 1.5 times larger than on the main sequence in the apparent separation range of 150 to 1800 AU (Reipurth & Zinnecker 1993). An infrared speckle interferometry survey of young stars in the Taurus star forming region, combined with direct imaging and lunar occultation observations, shows that approximately two times as many binaries are found in the apparent distance range of 18 to 1800 AU in the Taurus star-forming region than on the main sequence (Leinert et al. 1993 and Richichi et al. 1994). A similar study of the Taurus-Auriga and Ophiuchus-Scorpius star-forming regions by Ghez, Neugebauer & Matthews (1993), on the other hand, indicates a factor of four times as many pre-main sequence binaries than main sequence stars in the apparent separation range 16 to 252 AU. Their incompleteness corrections differ from those applied by the other groups, in that they attempt to account for missed companions fainter than the primary star by more than two K-band magnitudes (for a further discussion of the incompleteness corrections and pre-main sequence binary stars see Mathieu 1994). Mathieu (1992) reports on radial velocity measurements of pre-main sequence stars and finds that the proportion of these that are binaries with  $P < 100$  days (semi-major axis less than 0.5 AU if the mass of the system is  $1.4 M_\odot$ ) is indistinguishable from the proportion on the main sequence.

The above studies suggest that the distribution of orbital elements for very young systems differs from those on the main sequence. It would appear that the form of the period distribution evolves as well as the number of binaries. The two mechanisms by which orbital parameters of binary systems evolve are: 1) Interaction of the two proto stars with each other and with the proto stellar material that remains in the system (*pre-main sequence eigenevolution*), and 2) perturbations caused by close passages of other systems (*stimulated evolution*). The first mechanism becomes increasingly important for systems with smaller separation of components and will be most effective in systems that are younger than the evolution time of the circumproto-stellar material which is roughly  $10^5$  yrs. This mechanism implies a more or less continuous evolution of orbital parameters. The second mechanism, on the other hand, is effective on a time scale characterised by the rate of evolution of the cluster of young stars and is increasingly effective for systems with larger separation of components. The close passage of another system leads to abrupt changes of the orbital elements. In this paper we are concerned with stimulated evolution only.

Past simulations of interactions of binaries with single stars show that these can lead to ionisation of the binary, or to changes in orbital elements and exchange of stellar components. In particular, Heggie (1975) developed in detail the theoretical understanding of the reaction of binary stars to perturbations by other systems and highlights the complexity of the interactions of higher order systems, and Hills (1975) performed extensive numerical simulations of such processes. Both studies show that tightly bound binary stars tend to heat the field by gaining binding energy, whereas relatively wide systems are ionised rapidly without affecting the energy content of the surrounding field significantly.

In this paper we study the evolution of the distribution function of periods, eccentricities and mass ratios of a population of binary stars that are initially grouped in an aggregate. We adopt initial distributions that are consistent with all available pre-main sequence observational constraints. By fitting the final model distribution functions to observed Galactic field distributions of orbital elements we hope to learn something about the typical dynamical configuration when stars form (*inverse dynamical population synthesis*).

This paper is the first in a series of three papers which we refer to as K1 (this paper), K2 (Kroupa 1995a) and K3 (Kroupa 1995b). In K2 we discuss the properties of the model Galactic field star population under the hypothesis that the stars originate in the dominant mode cluster, and elaborate on the concept of eigenevolution. In K3 we discuss the observational consequences of the dynamical evolution of star-forming aggregates, making use of the simulations performed in papers K1 and K2.

The observational constraints on orbital parameters are discussed in Section 2. We introduce our models in Section 3. In Section 4 we perform inverse dynamical population synthesis to isolate a possible dominant mode of star-formation that may lead to the population of Galactic field stars. By correcting the main sequence distribution of orbital elements for the dynamical evolution in the dominant mode cluster we derive an initial period distribution in Section 5. In Section 6 we discuss our findings and Section 7 presents our conclusions.

## 2 THE OBSERVATIONAL CONSTRAINTS ON ORBITAL PARAMETERS

In Fig. 1 we reproduce the distribution of periods,  $f_P$ , for G, K and M dwarf systems. It is evident that  $f_P$  does not significantly depend on spectral type. The total proportion of late-type binary systems in the Galactic disk amounts to  $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ , which is a weighted average of the proportion of binaries among G, K and M dwarfs, respectively:  $f_G = 0.53 \pm 0.08$ ,  $f_K = 0.45 \pm 0.07$  and  $f_M = 0.42 \pm 0.09$  (see Leinert et al. 1993 and Fischer & Marcy 1992 for the compilation of these numbers – note that a weighted average is not a strictly correct estimate of  $f_{\text{tot}}$ , but suffices for our purpose;  $f_{\text{tot}}$  is defined by equation 2 below, see also equation 7). In the top panel of Fig. 2 we show the mass-ratio distribution for binary systems with a solar mass primary (Duquennoy & Mayor 1991). We correct this mass-ratio distribution for the bias that hampers spectroscopic surveys in that for a given orbital mass function not all inclinations are accessible to an observer (Mazeh & Goldberg 1992). To this end we add the long-period ( $\log_{10}P > 3.5$ ) mass ratio distribution of Duquennoy & Mayor (1991) and the corrected short period ( $\log_{10}P < 3.5$ ) mass ratio distribution of Mazeh et al. (1992, equations 1 and 2). The corrected mass ratio distribution is shown in the top panel of Fig. 2 as well as the expected mass ratio distribution for binaries with a solar mass primary if the secondaries were to follow the Galactic field star mass function. The G-dwarf binary star data show a depletion of small mass ratio systems. This discrepancy has also been pointed out by Kroupa & Tout (1992). In the middle panel of Fig. 2 we contrast the mass-ratio distribution for short-period systems (Mazeh et al. 1992) with the distribution for long-period systems (Duquennoy & Mayor 1991) after normalising both to unit area. It appears that both are fundamentally different. It is noteworthy that Goldberg & Mazeh (1994) find a mass ratio distribution for short period Pleiades binaries rather similar to that shown here, although the statistics is very poor. The CORAVEL study also shows that the eccentricities for G-type main sequence binaries with periods less than approximately 12 days are circularised. For  $11.6 < P < 1000$  days the eccentricity distribution is bell shaped with a mean eccentricity  $\bar{e} = 0.31 \pm 0.04$  whereas for  $P > 1000$  days Duquennoy & Mayor find the data are consistent with the thermal distribution (Section 3.2.1). These eccentricity distributions are shown in the bottom panel of Fig. 2. Both the mass-ratio as well as the eccentricity distributions undergo a significant change of character at  $\log_{10}P \approx 3$ . This point will be further discussed in K2.

The data that are currently available for pre-main sequence binaries are still somewhat scarce (for a more in depth review of the observations see Mathieu 1994). In Fig. 1 we compile the measured distributions

in  $\log_{10}P$  obtained from the separations of the components. The histograms for pre-main sequence binaries shown in Fig. 1 are obtained from Simon (1992), Leinert et al. (1993) and Richichi et al. (1994), and by rebinning the data assuming a mean stellar mass of  $1.4 M_\odot$ ,  $1 M_\odot$  and  $1.4 M_\odot$  published by Reipurth & Zinnecker (1993) and Ghez et al. (1993), respectively, into  $\Delta \log_{10}P = 1$  wide bins. Mathieu (1992) finds that the proportion of short period pre-main sequence binaries ( $\log_{10}P < 2$ ) is virtually the same as for main sequence binaries. Leinert et al. (1993) discuss the distribution of brightness ratios for their sample of pre-main sequence binaries. Although the mapping to a mass ratio distribution is very uncertain they find that their data are consistent with random pairings from the KTG(1.3) mass function, which increases as a power-law with decreasing mass (equation 1 below). This mass function provides a better fit to their data than a mass function which flattens or decreases below about  $0.3 M_\odot$ , and which fits the mass-ratio distribution of solar-type binaries (Duquennoy & Mayor 1991).

Comparison of the period distribution of pre-main sequence binaries with the period distribution of main sequence binaries with a G, K and M star primary suggests evolution of the distributions. Comparing the main sequence data with the pre-main sequence data shown in Fig. 1 we must keep in mind that triple and quadruple systems are counted by Duquennoy & Mayor (1991) as two and three orbits, respectively, in the former and that the latter data are derived from apparent separations on the sky. However, the discussion in this paper is not affected by the different nature of the two samples.

The pre-main sequence binary star data were obtained from observations of distributed star forming regions (Taurus–Auriga and Ophiuchus–Scorpius). Individual star-forming regions might lead to distributions of orbital parameters that differ from each other, as discussed by Reipurth & Zinnecker (1993), Leinert et al. (1993), Ghez et al. (1993), Mathieu (1992), Simon (1992) and Durisen & Sterzik (1994). However, variation of the binary star distribution at birth between different star-forming regions is suggested but by no means established by the data at present. Furthermore, we stress that even if different binary proportions and period distributions are observed in different star forming regions it must first be established whether the differences are not due to different dynamical ages of the regions rather than being the result of different star-formation conditions, as we discuss in greater length in Section 6.3 and in K3.

### 3 MODELS

In this section a brief description of the numerical tool is given, and the set of numerical experiments performed in Section 4 are detailed.

#### 3.1 The N-body program

We need to integrate stellar orbits in a system with a few hundred binary stars with the principal aim of studying the evolution of their orbital elements. The processes responsible for the change of orbital parameters of a binary system operate on a timescale that is many orders of magnitudes less than the timescale over which an aggregate of stars evolves. Both timescales are important for our work.

Aarseth has been developing a N-body code for the direct integration of the orbits of stars in a cluster (Aarseth 1994, see also Aarseth 1992). We make use of his *nbody5* code in which the regularisation of the equations of motion of more than two point stars in close proximity (Mikkola & Aarseth 1990, 1993) makes the computational task we are faced with possible. We model a standard Galactic tidal field, with Oort's constants  $A = 14.4 \text{ km sec}^{-1} \text{ kpc}^{-1}$ , and  $B = -12 \text{ km sec}^{-1} \text{ kpc}^{-1}$ , and a local mass density of  $0.11 M_\odot \text{ pc}^{-3}$ . We retain all stars during the simulations irrespective of whether stars are bound to the aggregate or not.

To identify all bound binary systems we supplement *nbody5* by an extensive data reduction program which searches all position and velocity vectors (with respect to the local standard of rest) of the stars output by *nbody5* at some prechosen time intervals to find the binaries. Each star  $i$  is paired with every other star  $j$  to find that pair for star  $i$  with the smallest separation of components. All pairs  $i$  thus found are compared to exclude repetition of components. The binding energy for each surviving pair is computed, and if negative, this pair is classified as a bound binary system, for which orbital elements are computed.

The orbital elements thus found need to be checked for stability against numerically induced evolution. This is achieved by comparing the eccentricity–period data at different time intervals for an aggregate with half mass radius of 31 pc consisting of two hundred binaries. Such a large half mass radius ensures that the

inter-binary distance is large so that stimulated evolution of the orbital parameters is negligible. Binary systems with periods smaller than  $10^8$  days show no changes in their orbital parameters over a time span of 1.3 Gyr. Systems with periods larger than this are subject to perturbation by the Galactic tidal field and suffer changes in eccentricity that increase with increasing period. Systems that initially had a period larger than approximately  $10^{10.3}$  days (corresponding to a semi-major axis of approximately 0.6 pc for a system mass of  $0.64 M_\odot$ ) are disrupted. The widest binary systems known have a separation of components of 0.1 pc (Close, Richer & Crabtree 1990) and we are thus satisfied that the N-body program correctly integrates the motion of stars in isolated binaries.

### 3.2 Initial parameters

#### 3.2.1 Binary stars

We concentrate on low-mass stars with a mass between 0.1 and  $1.1 M_\odot$  because these make up the bulk of the stellar population in the Galactic disk. It is for these that we have the best measurements of binary star distribution functions and we do not have to take into account post-main sequence evolution, when mass loss through stellar winds and supernova explosions complicate the dynamics of a stellar aggregate. Also, the recent studies on duality in star-forming regions concentrate on T-Tauri type stars. The masses are chosen from the following mass-generating function (Kroupa et al. 1993):

$$m(X) = 0.08 + \frac{\gamma_1 X^{\gamma_2} + \gamma_3 X^{\gamma_4}}{(1 - X)^{0.58}}, \quad (1a)$$

where  $X$  is distributed uniformly in the range 0 to 1,  $m$  is the stellar mass in solar units and  $\gamma_1 = 0.19$ ,  $\gamma_2 = 1.55$ ,  $\gamma_3 = 0.050$  and  $\gamma_4 = 0.6$  if  $\alpha_1 = 1.3$  in equation 1b. Equation 1 leads to a stellar mass spectrum in the local Galactic disk, derived from both nearby and deep star count data. After correcting for post-main sequence stellar evolution it can be approximated conveniently by a three-part power law initial mass function:

$$\xi(m) = 0.087 \begin{cases} 0.5^{\alpha_1} m^{-\alpha_1}, & \text{if } 0.08 \leq m < 0.5; \\ 0.5^{2.2} m^{-2.2}, & \text{if } 0.5 \leq m < 1.0, \\ 0.5^{2.2} m^{-2.7}, & \text{if } 1.0 \leq m < \infty, \end{cases} \quad (1b)$$

where  $\xi(m) dm$  is the number of stars per  $\text{pc}^3$  in the mass range  $m$  to  $m + dm$ . We refer to this spectrum of stellar masses as the KTG( $\alpha_1$ ) mass function and adopt  $\alpha_1 = 1.3$ . The mean stellar mass of our population is  $0.32 M_\odot$ . Stars more massive than  $1.1 M_\odot$  contribute 8 per cent by number and 35 (39) per cent by mass to the stellar population if the most massive star born has a mass of 10 (50)  $M_\odot$ .

A critical analysis of the methods used to derive mass-ratio distributions by Tout (1991) suggests that the simplest of selection effects can account for most structure claimed in the observed mass-ratio distribution. The lack of evidence for significant correlation is further discussed by Kroupa et al. (1993). As discussed in the introduction, Leinert et al. (1993) find evidence that the component masses of pre-main sequence stars are chosen at random from equation 1. Consequently we pair the stellar masses generated from equation 1a at random to form a binary star population that has an uncorrelated mass ratio distribution  $f_q(q)$  at birth (shown in fig. 12 in K2), where  $f_q(q) dq$  is the proportion of systems with a mass ratio in the range  $q$  to  $q + dq$ . Here  $q = m_2/m_1 \leq 1$ , where  $m_1$  and  $m_2$  are the mass of the primary and secondary component, respectively.

The initial orbital parameters of a freshly hatched binary star population are completely unknown. Because the binary proportion is high on the main sequence and even higher in star-forming regions we assume that all stars form as binaries, *i.e.* we set  $f_{\text{tot}} = 1$  initially. We define the overall fraction of binaries at time  $t$  as

$$f_{\text{tot}}(t) = \frac{N_{\text{bin}}(t)}{N_{\text{sing}}(t) + N_{\text{bin}}(t)}, \quad (2)$$

where  $N_{\text{bin}}(t)$  and  $N_{\text{sing}}(t)$  are the number of multiple systems and single stars, respectively, and the denominator is the number of all systems (*c.f.* Reipurth & Zinnecker 1993).

The distribution function of semi-major axes,  $a$ , is approximated rather well by a Gaussian in  $\log_{10}a$  for binaries on the main sequence (Fig. 1). Our assumption here is that the initial distribution is flat, straddling the peak at  $a \approx 30$  AU in the main-sequence distribution of semi-major axes. The generating function is

$$a(X) = a_{\min} 10^{X \log_{10}\left(\frac{a_{\max}}{a_{\min}}\right)}, \quad (3a)$$

where  $a_{\max}$  and  $a_{\min}$  are the maximum and minimum semi-major axes we wish to include and  $X \in [0,1]$  is a uniform random variate. We choose  $a_{\max} = 1690$  AU and  $a_{\min} = 1.69$  AU. The distribution function is

$$f_a(\log_{10}a) = [\log_{10}(a_{\max}) - \log_{10}(a_{\min})]^{-1}, \quad (3b)$$

where  $dn(\log_{10}a) = N_{\text{bin}} f_a(\log_{10}a) d\log_{10}a$  is the number of orbits with semi-major axis in the range  $\log_{10}a$  to  $\log_{10}a + d\log_{10}a$ . This distribution is equivalent to a flat  $\log_{10}P$  distribution with  $10^3 \leq \log_{10}P \leq 10^{7.5}$  (for a mean system mass of  $0.64 M_{\odot}$ ) which is consistent with the pre-main sequence data (Fig. 1).

We assume the initial eccentricity distribution function represents a population in statistical equilibrium. The generating function is

$$e(X) = \sqrt{X}, \quad (4a)$$

which has the thermal distribution function

$$f_e(e) = 2e. \quad (4b)$$

The number of systems with eccentricity in the range  $e$  to  $e + de$  is  $dn(e) = N_{\text{bin}} f_e(e) de$  (see for example Heggie 1975). This dynamically relaxed distribution has the advantage that most binary systems have large eccentricity in agreement with recent studies of star formation (Boss 1988, Pringle 1989, Bonnell & Bastien 1992, Elmegreen 1993, Burkert & Bodenheimer 1993), although theory cannot predict orbital elements because computations have to be stopped before the stars have accreted most of the natal material. The results of this paper are not sensitive to the form of  $f_e(e)$  (see K2).

In summary, we assume the initial distribution function can be separated:

$$D(P, e, q) = f_P(\log_{10}P) \times f_q(q) \times f_e(e), \quad (5)$$

where  $f_P$  is the distribution of  $\log_{10}P$  and is obtained from equation (3b) using Kepler's equation.

### 3.2.2 The stellar aggregates

Stars in the Galaxy are formed in groups in molecular clouds but it is not clear which parameters of these groups are typical for field stars. In Section 4 we model the evolution of a number of initial aggregates of stars to later identify those that lead most closely to the distribution of binary properties observed on the main sequence.

To cover a range of possibilities we adopt Plummer models with various radii as given in Table 1. Generating functions for Plummer spheres can be found in Aarseth, Hénon & Wielen (1974). We assume the aggregates are in virial equilibrium, i.e. we choose  $Q = -K/W = 0.5$ , where  $W$  is the total binding energy of the cluster and  $K$  is the sum of the kinetic energies of all centre of masses in the cluster. In our simulations stars are treated as point particles.

We emphasise that we choose Plummer models merely for our convenience and that we do not insist that stars form in such systems. Our choice of aggregate can only be a first try. We must keep in mind that the early internal dynamics of a cluster of proto stars is dominated by a rapidly changing potential because after the stars begin to form a large proportion of the mass, which remains as gas and dust, of most star-forming systems is removed within of the order of  $10^7$  years (Mathieu 1986, Battinelli & Capuzzo-Dolcetta 1991). Most groups of young stars may become gravitationally unbound on this timescale (we return to this point in Section 6.4). Also, we expect that about 8 per cent of all stars have masses larger than  $1.1 M_{\odot}$ . By excluding these and the changing background potential we introduce a stellar mass dependent bias because mass segregation leads to G dwarfs (which are the most massive stars in our simulations) spending, on average, more time near the cluster centre and are thus subject to stimulated evolution over a longer time

scale than the less massive stars. The dynamics in a real cluster of young stars is affected by mass loss from evolving stars and supernovae. Finally, the finite size of stars leads to (rare) stellar collisions and mergers, as well as removal of kinetic energy through tidal dissipation in close fly-by events. These complications do not, however, affect the results presented in this paper.

In Table 1, Column 1 lists the paper which discusses the simulations (K1=this paper, K2=Kroupa 1995a, K3=Kroupa 1995b). Column 2 lists the initial half mass radius,  $R_{0.5}$ . We choose four initial values of  $R_{0.5}$  which span the range 0.077 pc to 2.53 pc. The lower limit corresponds to tightly clustered star formation (e.g. Trapezium Cluster), and the upper limit approximates isolated star formation (e.g. Taurus–Auriga). Columns 3 and 4 contain the initial number of binary systems and the number of systems composed of single stars, respectively. The aggregate mass is in all cases  $128 M_\odot$ . We do not expect that each star-forming event produces exactly 400 stars, but we consider this assumption to be an adequate first guess. In this context it is of interest to note that Battinelli, Brandimarti & Capuzzo-Dolcetta (1994) find that the mass function of open clusters peaks at a mass of  $126 M_\odot$ , which is consistent with the IRAS selected sample of embedded clusters obtained by Carpenter et al. (1993). Columns 5, 6 and 7 contain, respectively, the initial and final proportion of binaries and the initial number density  $n$  of stars within a sphere with a radius of 2 pc centred on the number density maximum,  $n_c$ , the initial value of which is listed in Column 8. Our maximum central number density is comparable to that observed in the Trapezium Cluster ( $\approx 4.7 \times 10^4$  stars pc $^{-3}$ , McCaughrean & Stauffer 1994). Our smallest central number density is an adequate approximation of isolated star formation (e.g. Taurus–Auriga). Column 9 gives the initial average velocity dispersion for each system and Columns 10 and 11 list the initial crossing time,  $T_{\text{cr}} = 2 R_{0.5}/\sigma$ , and median relaxation time,  $T_{\text{relax}} \propto R_{0.5}^{3/2} N_{\text{bin}}^{1/2} / \log_{10}(0.8 N_{\text{bin}})$  (Spitzer & Hart 1971), respectively. The number of simulations per cluster ( $\geq 3$  to increase statistical significance) with different initial random number seeds are given in Column 12. In all subsequent discussions the mean values of the  $N_{\text{run}}$  runs are used for the relevant parameters, and their quoted uncertainties are standard deviations of the mean. The mean lifetime,  $\tau_{0.1}$ , of each cluster is given in Column 13 and is defined in K3. It is the time required for the central number density to drop below 0.1 stars pc $^{-3}$ , and signifies complete disintegration. We shall refer to ‘final’ distributions as distributions evaluated after aggregate disintegration at  $t = 1$  Gyr. Finally, in Column 14 we list the ratio of the initial cluster binding energy,  $W$ , and of the binding energy of a binary,  $E_b$ , with  $1 M_\odot$  companions and shortest period occurring in the birth period distributions used (i.e. in K2 prior to eigenevolution). The single-star runs ( $N_{\text{bin}} = 0$ ), which are of academic interest only, are used for comparison with the evolution of the realistic aggregates consisting initially only of binaries.

**Table 1.** Initial stellar aggregates studied in papers p.K1, K2 and K3

p.K	$R_{0.5}$	$N_{\text{bin}}$	$N_{\text{sing}}$	$f_{\text{tot}}$	$f_{\text{tot}}$	$n$	$\log_{10}(n_c)$	$\sigma$	$T_{\text{cr}}$	$T_{\text{relax}}$	$N_{\text{run}}$	$\tau_{0.1}$	$E_b/W$
	pc			$t = 0$	$t = 1$ Gyr	stars	stars	km	Myrs	Myrs		Myrs	
						pc $^{-3}$	pc $^{-3}$	sec $^{-1}$					
1,3	0.08	200	0	1	$0.269 \pm 0.015$	12	5.6	1.7	0.094	0.30	5	$655 \pm 137$	0.7
1,3	0.25	200	0	1	$0.400 \pm 0.020$	12	4.1	0.9	0.54	1.8	5	$696 \pm 135$	2
1,3	0.77	200	0	1	$0.610 \pm 0.037$	11	2.7	0.5	3.0	9.5	5	$748 \pm 114$	7
1,3	2.53	200	0	1	$0.823 \pm 0.018$	5	1.1	0.3	17	56	5	$693 \pm 127$	20
1,3	0.08	0	400	0	$0.021 \pm 0.006$	12	5.6	1.7	0.094	0.30	3	$736 \pm 178$	–
1,3	0.25	0	400	0	$0.020 \pm 0.006$	12	4.1	0.9	0.54	1.8	3	$732 \pm 138$	–
1	0.77	200	0	1	$0.553 \pm 0.048$	11	2.7	0.5	2.9	9.5	5	$705 \pm 247$	490
1	0.77	200	0	1	$0.416 \pm 0.042$	11	2.7	0.5	2.9	9.5	5	$686 \pm 85$	490
2,3	0.85	200	0	1	$0.480 \pm 0.032$	10	2.5	0.5	3.5	11	20	$740 \pm 150$	120

## 4 INVERSE DYNAMICAL POPULATION SYNTHESIS

In this section we show that the dynamical properties of stellar systems in the Galactic field can be derived if the majority of stars form in a characteristic aggregate.

### 4.1 The dynamical properties of stellar systems

Assume we know that star formation produces stellar systems with a distribution of birth *dynamical properties*,  $\mathcal{D}(\xi(m), D(P, e, q), N, R_{0.5})$ , which is the distribution of initial mass functions, of period, eccentricity and mass-ratio distributions and of the number of stars forming in a volume characterised by some half-mass radius. The Galactic field population is given by the time evolved integral of  $\mathcal{D}$  over all star forming events ever to have occurred. Thus, if we know  $\mathcal{D}$ , we can compute the dynamical properties (stellar masses, orbital parameters, multiplicities) of the stellar population of the Galactic field under some assumed star formation history.

However,  $\mathcal{D}$  is completely unknown. We invert the problem here by investigating if a representative set  $\xi(m), D(P, e, q), N$  and  $R_{0.5}$  which represents the most common mode, or average of  $\mathcal{D}$ , can be deduced from the currently observed stellar dynamical properties in the Galactic disc. Kroupa et al. (1993) have determined  $\xi(m)$  (equation 1) and in Section 3.2.1 we adopt a  $D(P, e, q)$  which is consistent with all available pre-main sequence data. In Section 3.2.2 we argue that  $N_{\text{bin}} = 200$  is a reasonable first guess, and we are left to estimate a characteristic, or representative,  $R_{0.5}$ .

### 4.2 Stimulated evolution of orbital parameters of binary stars

As mentioned in the introduction, past work has established that binary systems are ionised at a rate which is a function of the ratio of the internal binding energy of the binary and the kinetic energy, or temperature, of the surrounding field population. In fig. 5 of K3 we discuss the initial and final distributions of binary star binding energies and centre of mass kinetic energies for each aggregate. Preferentially those binary systems are ionised that are relatively least bound.

The overall evolution of the binary star population is best summarised by studying the evolution of the proportion of binaries,  $f_{\text{tot}}(t)$ , given by equation 2. In Fig. 3 we plot  $f_{\text{tot}}$  as a function of time,  $t$ , for the first four aggregates listed in Table 1. We also show  $f_{\text{tot}}$  for the additional simulations with no primordial binaries ( $N_{\text{bin}} = 0$ ). From Fig. 3 we deduce that the Galactic disk main sequence stellar population, for which  $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ , is best reproduced if  $0.25 \text{ pc} < R_{0.5} < 0.8 \text{ pc}$ . Three further points are displayed by Fig. 3: (i) The rate of evolution of  $f_{\text{tot}}$ , and for that matter of all binary star orbital parameters, is largest while the cluster is dynamically young, but proceeds much slower in terms of absolute time in clusters with larger initial size. Thus, for example, we expect a very young (one Myr old) embedded cluster with high central number density (about  $10^5 \text{ stars pc}^{-3}$ ) to have a significantly depleted binary star population. An example of such a cluster is the Trapezium Cluster which we will discuss in greater detail in K3. (ii) The final proportion of binaries is smaller when the initial number density is larger. The reduction of the proportion of multiple systems with increasing number of systems per cloud suggested by fig.4 of Reipurth & Zinnecker (1993) is interesting in this context. (iii) The proportion of binaries formed by capture ( $f_{\text{tot}} \approx 0.02$ ) is insignificant compared to the observed proportion ( $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ ) which is merely confirmation of the long established understanding that dissipationless capture is not a viable formation mechanism for binary systems (see Bodenheimer et al. 1993 and references therein), even if initial subclustering, which enhances the proportion of binaries formed by capture (Aarseth & Hills 1972), is taken into account.

The binding energy of a binary system can be written

$$-E_{\text{bin}} = -G \frac{m_1^2}{2a} q. \quad (6)$$

We thus have  $E_{\text{bin}} \propto q$  and we immediately see that in a cluster the mass ratio distribution must be depleted such that systems with small  $q$  are ionised at a larger rate than those with  $q \approx 1$ . In Fig. 4 we show the mass function of secondaries in systems with a primary mass  $0.85 M_{\odot} < m_1 < 1.1 M_{\odot}$  for each of the first four aggregates listed in Table 1 initially and after aggregate dissolution. The increasing bias towards  $q = 1$

with decreasing initial aggregate size is apparent, and comparison with the G-dwarf main sequence binary star distribution suggests that the observed distribution is best reproduced if  $R_{0.5} \approx 0.8$  pc.

The distribution of periods is expected to be more depleted at long periods than at short periods. In Fig. 5 we show the initial and final distributions of periods of our binary star population. Normalisation of the histograms is obtained from

$$f_{P,i}(t) = \frac{N_{\text{bin},i}(t)}{N_{\text{bin}}(t) + N_{\text{sing}}(t)} \quad (7a)$$

with

$$\sum_{\text{all bins } i} f_{P,i}(t) = f_{\text{tot}}(t), \quad (7b)$$

where  $N_{\text{bin},i}$  is the number of orbits in the  $i$ th  $\log_{10}P$  bin, and the denominator is as in equation 2. On comparison with the observed period distribution (Fig. 5) we deduce that the main sequence period distribution is best reproduced if  $R_{0.5} \approx 0.8$  pc. From Fig. 5 we also see that the number of orbits with  $\log_{10}P < 3$  and  $\log_{10}P > 7.5$  does not increase significantly in any of the aggregates. This lack of hardening and softening of binaries comes about because the lifetimes of the aggregates studied here are too short (*total* disintegration after about 700 Myrs, see K3).

The eccentricity distribution remains thermal (equation 4) at all times and in all clusters.

#### 4.3 Dynamically equivalent aggregates

Given our assumptions which are consistent with all available observational constraints (Section 3.2.1) we have shown that the dynamical properties of Galactic field systems can be derived if most stars form in an aggregate with  $(N_{\text{bin}}, R_{0.5}) \approx (200, 0.8$  pc).

We postulate that most stars may have formed in aggregates which are dynamically equivalent to this *dominant mode cluster*.

We define a stellar aggregate or cluster to be *dynamically equivalent* to the dominant mode cluster if stimulated evolution evolves the initial population of stellar systems (with initial dynamical properties as defined in Section 3.2.1 and equation 11 below) to a population which has final stellar dynamical properties that are consistent with the observed properties of Galactic field stellar systems (see also section 4.2 in K3).

### 5 THE INITIAL PERIOD DISTRIBUTION

Star formation theory is not currently in a state that allows prediction of either the proportion of multiple systems formed, or the distribution of periods of the primordial binary star population (see e.g. Bonnell 1994). Any clues pertaining to the primordial orbital parameters of a binary star population are thus very valuable. In Section 4.2 we have seen that even very simple models of the initial period, eccentricity and mass-ratio distribution (which are consistent with the available observational data) lead to encouraging results given they are subject to the internal dynamics of a stellar aggregate.

Investigating Fig. 5 we conclude that it must be possible, by correcting the observed main sequence period distribution for stimulated evolution, to identify an initial period distribution that evolves to the observed main sequence distribution. We proceed as follows: The simulations discussed in Section 4.2 are our zeroth order iteration. In Sections 5.1 and 5.2 we go to the first and second order iteration, respectively which are the likely range of the initial period distribution.

#### 5.1 The first iteration

We calculate the orbit depletion function

$$c_{P,i}(t) = \frac{f_{P,i}(t)}{f_{P,i}(0)}, \quad (8)$$

where  $f_{P,i}(t)$  is the proportion of orbits in the  $i^{\text{th}}$   $\log_{10}P$  bin at time  $t$  (equation 7). The ratio  $c_{P,i}(t)$  measures the depletion of orbits in the  $i^{\text{th}}$  period bin. The final distribution is plotted in Fig. 6. Hardened binaries now appear at  $\log_{10}P < 3$  ( $c_{P,i} > 1$ ). If we denote the observed main sequence period distribution (Fig. 1) by  $f_{P,\text{ms},i}$  then the initial period distribution is given by

$$f_{P,\text{in},i} = f_{P,\text{ms},i} c_{P,i}(1 \text{ Gyr})^{-1} \quad (9a)$$

with the constraint

$$\int_{\log_{10}P_{\min}}^{\log_{10}P_{\max}} f_{P,\text{in}} d\log_{10}P = 1 \quad (9b)$$

which becomes a sum for the discrete case (equation 9a). In Fig. 7  $f_{P,\text{in},i}$  is plotted for the first four aggregates listed in Table 1. From Fig. 7 we deduce that  $f_{P,\text{in},i} \propto \log_{10}P$  approximately. We adopt as our first try the distribution function

$$f_{P,\text{in}}^{(1)} = \beta [\log_{10}(P) - \log_{10}(P_{\min})] \quad (10a)$$

with the period generating function

$$\log_{10}P(X) = \left(\frac{2}{\beta}X\right)^{\frac{1}{2}} + \log_{10}P_{\min} \quad (10b)$$

where  $X \in [0, 1]$  is a uniform random variate. Fig. 7 suggests  $\log_{10}P_{\min} \approx 2$ . However, this choice would imply a deficit in main sequence orbits for  $\log_{10}P < 3$ . This deficit of orbits would not be made up by stimulated evolution as hardening of binary orbits is not efficient enough (Section 4 and K2). We address the question as to whether there is a  $P_{\min} > 1$  day in K2 and for the present adopt  $\log_{10}P_{\min} = 0$ .

The slope  $\beta$  and maximum period  $P_{\max}(X = 1)$  are tabulated in Table 2. With our above choice for  $\log_{10}P_{\min}$  the linear approximation given by equation (10a) only allows a rough measurement of  $\beta$  for the aggregates with  $R_{0.5} = 0.08, 0.25, 2.53$  pc (see Fig. 7).

**Table 2.** Preliminary initial period distribution (equation 10)

$R_{0.5}$ (pc)	$\beta$	$\log_{10}P_{\max}$ (days)
0.08	$\approx 0.058$	5.9
0.25	$\approx 0.042$	6.9
0.77	0.034	7.67
2.53	$\approx 0.020$	10

From Table 2 we deduce that aggregates approximately with  $R_{0.5} \geq 0.77$  pc require  $\beta$  that imply  $P_{\max} > 10^{7.5}$  days which is a *necessary* condition for agreement with the observational data. Also, from our discussion of the mass ratio distribution in Section 4.2 we require  $R_{0.5} < 2.5$  pc and we now adopt  $R_{0.5} = 0.77$  pc,  $\beta = 0.034$  and  $P_{\max} = 10^{7.67}$  days as best approximating all observational constraints.

Thus we find that in order to be able to model the full range of orbital periods observed and the observed mass ratio distribution of long-period G dwarf binaries we require  $R_{0.5} \approx 0.8$  pc.

We perform  $N_{\text{run}} = 5$  runs of an aggregate with  $R_{0.5} = 0.77$  pc (the first cluster listed under p.K1 in Table 1). The final period distribution is shown in Fig. 8a. Comparing with the main sequence data we find that it is somewhat too large for  $\log_{10}P > 5$  and falls too steeply for  $\log_{10}P > 7$ . The final proportion of binary systems is  $f_{\text{tot}}^{(1)} = 0.55 \pm 0.05$  (cf.  $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ ).

It is highly improbable that the initial period distribution does in fact have the form of equation 10 although we cannot reject the resulting model main sequence distribution with high confidence. In particular, it is worthwhile to point out that Close et al. (1990) compiled a complete sample of wide binaries in the solar neighbourhood and found systems with separation of components as large as 0.1 pc. This is equivalent

to  $\log_{10}P = 7.5$  if the system mass is  $1.4 M_\odot$ . A power law decay of  $f_{P,\text{ms}}$  with increasing period cannot be rejected though, and Duquennoy & Mayor (1991) find evidence that systems with periods as large as  $10^9$  days exist. As equation 10 implies a sharp fall-off of the simulated  $f_{P,\text{ms}}$  for  $\log_{10}P > 7$  we expect the initial period distribution,  $f_{P,\text{in}}$ , must have  $\log_{10}P_{\text{max}} > 7.67$ . This requires a flattening at large periods given constraint (9b).

## 5.2 The second iteration

A distribution function which fulfills the requirement raised at the end of Section 5.1 and which is also easily integrable is

$$f_{P,\text{in}}^{(2)} = \eta \frac{(\log_{10}P - \log_{10}P_{\text{min}})}{\delta + (\log_{10}P - \log_{10}P_{\text{min}})^2} \quad (11a)$$

with the period generating function

$$\log_{10}P(X) = \log_{10}P_{\text{min}} + \left[ \delta \left( e^{\frac{2X}{\eta}} - 1 \right) \right]^{\frac{1}{2}} \quad (11b)$$

where  $X \in [0, 1]$  is a uniform random variate. For our second iteration we also assume  $\log_{10}P_{\text{min}} = 0$ . Adjustment of the parameters such that the slope of the distribution at small periods is approximately the same as for  $f_{P,\text{in}}^{(1)}$  and  $\log_{10}P_{\text{max}} > 7.5$  leads us to adopt  $\eta = 3.50$  and  $\delta = 100$  with  $\log_{10}P_{\text{max}} = 8.78$  as our second iteration.

The results of five simulations of a cluster with  $R_{0.5} = 0.77$  pc (second cluster listed in Table 1 under p.K1) are shown in Fig. 8b. The final proportion of binary systems is  $f_{\text{tot}}^{(2)} = 0.42 \pm 0.04$  (cf.  $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ ).

## 5.3 Result

The comparison of Figs. 8a and 8b, and  $f_{\text{tot}}^{(1)}$  and  $f_{\text{tot}}^{(2)}$  with  $f_{\text{tot}}^{\text{obs}}$ , shows that  $f_{P,\text{in}}^{(1)}$  and  $f_{P,\text{in}}^{(2)}$  are probably close to being upper and lower bounds of the initial period distribution.

We have found that in order to be able to model the full range of orbital periods observed we require  $R_{0.5} \approx 0.8$  pc, in agreement with our conclusion in Section 4.

## 6 DISCUSSION

### 6.1 Clustered star formation as the dominant mode of star formation?

Perhaps the most interesting insight we have gained here is that our inverse dynamical population synthesis suggests that clustered star formation may be the dominant mode rather than isolated or distributed star formation. We have been able to obtain this insight by using the dynamical properties of observed stellar systems in the Galactic disk as tracers of the evolutionary histories of the systems, together with simple assumptions about the initial dynamical properties that are consistent with all observational constraints. Our finding is thus independent from and much more general than imaging surveys of star forming regions which are restricted to individual molecular clouds and cannot at present detect pre-main sequence stars with masses lower than about  $0.5 M_\odot$ .

However, it is reassuring that surveys of various star forming regions find a similar result. Lada & Lada (1991) list in their table 1 the physical characteristics of the embedded clusters in the L1630 molecular cloud in the Orion complex. An imaging survey at near-infrared wavelengths which can detect sources of a few tenths solar mass found 627 sources. Of these 80 per cent are located in two embedded clusters with radii of 0.86 and 0.88 pc, and the rest are located in two embedded clusters with radii of 0.59 and 0.30 pc. Carpenter et al. (1993) image 20 bright IRAS sources at near-infrared wavebands and find in all cases that embedded clusters of low mass stars are associated with the IRAS sources (usually one or two OB stars). Their radii are typically roughly 0.5 pc with a typical stellar number density of  $70 \text{ stars pc}^{-3}$ . These values are lower limits because the faint stars are not detected, but demonstrate that these specially selected systems are similar to our dominant mode cluster. On the other hand, the near-infrared survey of the L1641 molecular cloud (Strom, Strom & Merrill 1993) finds a significant distributed population of about 1500 about 5 – 7 Myrs old

stars, seven aggregates consisting of 10–50 stars with volume densities of roughly  $10^2$  stars pc $^{-3}$  and ages less than 1 Myr, and one new embedded cluster with about 150 stars and a volume density of  $10^2$  stars pc $^{-3}$ . Thus, direct imaging surveys cannot at present verify whether a single mode (clustered or distributed) of star formation predominates. In addition, we point out that the characteristics of the binary star population are unknown for any of the above surveyed star-forming systems. If our conclusions are correct then the binary proportion must be very high in embedded clusters, and the mass-ratio distribution of the young binaries must be approximately uncorrelated. The details will be a function of the dynamical age of each individual stellar aggregate.

Radial velocity and proper motion measurements of the distributed population in L1641 may shed light on the possibility that some or most of the distributed stars have migrated from aggregates that are now dissolved. However, the spatial motion of a young star after leaving the aggregate will probably change significantly because the gravitational potential of the molecular cloud is not likely to be smooth. In K3 we argue that a distributed population of young stellar objects is obtained after dissolution of birth aggregates because most systems will have velocities that are much smaller than the escape velocity from the molecular cloud. The proportion of binaries and their orbital parameters is a potentially powerful discriminant for the origin of a distributed population (K3).

## 6.2 Concerning the stellar mass function...

Our finding that most stars may have originated from aggregates that are dynamically equivalent to our dominant mode cluster rests in part on our assumed stellar mass function and the discrepancy with the mass-ratio distribution of G dwarf main sequence binary systems (Kroupa & Tout 1992). Although we are confident that the KTG(1.3) mass function (equation 1) is a good approximation to the true initial stellar mass function of Galactic field stars (Kroupa 1995c) we now contemplate a different form to test the robustness of our conclusions.

For example the mass function could flatten below about  $0.5 M_\odot$  or decrease below about  $0.2 M_\odot$ , i.e. it could have the same form as the mass function of secondaries in G dwarf binaries with  $\log_{10}P > 3$ . In this case we would have to argue that the stellar luminosity function for nearby stars does not reflect the true Galactic field luminosity function for single stars (see Kroupa et al. 1993, Kroupa 1995c), and that the photometric luminosity functions are the correct single star luminosity functions, as was assumed for example by Kroupa, Tout & Gilmore (1990) in their initial analysis of structure in the mass–luminosity relation.

Ignoring this, and the strong exclusion of such a mass function by Kroupa et al. (1993, their section 9), we would find that our inverse dynamical population synthesis would identify as the most common mode of star formation an aggregate similar to the  $R_{0.5} = 2.53$  pc model listed in Table 1 because in this case stimulated evolution must be kept to a minimum in order not to deplete the secondary mass function in G dwarf binaries significantly (Fig. 4). This mode corresponds roughly to the isolated or distributed mode in star formation with a central (i.e. maximum) initial density of approximately  $13$  stars pc $^{-3}$ . However, we would then find a discrepancy with the observed proportion of main sequence binaries ( $f_{\text{tot}}^{\text{obs}} = 0.47 \pm 0.05$ , Fig. 3) unless only about 50 per cent of all systems are binaries at birth. This is in contradiction with the observational evidence that the initial proportion of binaries is close to unity, whereby we stress that this observational evidence is strictly valid only for the distributed mode of star formation. Thus, following this line of argument we arrive at a contradiction (see also Section 6.5).

The KTG( $\alpha_1$ ) mass function may have a different  $\alpha_1$ . Star count data constrain  $\alpha_1$  to lie in the 95 per cent confidence range  $0.70 - 1.85$  (Kroupa et al. 1993). Disregarding this finding for illustrative purposes only and allowing  $\alpha_1 > 1.3$  our simulations would show that we need  $R_{0.5} < 0.8$  pc to force the final mass-ratio distribution of solar-type binaries to have the same shape as the observed distribution, as can be inferred by consulting Fig. 4. In this case, however, we would ionise too many binary systems implying  $f_{\text{tot}} < f_{\text{tot}}^{\text{obs}}$ . Alternatively, if  $\alpha_1 < 1.3$  ( $\alpha_1 < 0$  is the flat or decreasing mass function we have already excluded above) then we require  $R_{0.5} > 0.8$  pc in order not to evolve the mass ratio distribution of solar type binaries too much. This, however, implies  $f_{\text{tot}} > f_{\text{tot}}^{\text{obs}}$ .

While firm constraints on  $\alpha_1$  and  $R_{0.5}$  are not possible using these arguments because the observational constraints are still too poor, our gedanken experiment does suggest that there may exist a relationship between  $R_{0.5}$  and  $\alpha_1$ , i.e. between the stellar number density at birth and the initial mass function. This

line of thought implies that there may exist variation of the stellar mass function in different dynamical birth configurations. Podsiadlowski & Price (1992) obtain a qualitatively similar result. They suggest that the form of the mass function may be determined (at least in part, given their simple physical model) by a competition between accretion and proto-stellar collision rates. An insightful discussion of the physical factors which might influence the spectrum of stellar masses formed can also be found in Zinnecker, McCaughrean & Wilking (1993).

Returning to our line of thought, three distinct modes of star formation may have been identified (although it is likely that they form a continuous sequence): very low star formation efficiencies (the distributed mode), low star formation efficiencies (embedded clusters, i.e. the alleged dominant mode, and Galactic clusters with somewhat higher efficiencies) and high efficiencies (Globular clusters). Here high efficiency is equivalent to a deep potential well of the star-forming region so that only a relatively small part of the original gas can be blown out. The variations of the densities within any one of these modes may be too small to imply significant variations in the initial mass function. Thus the stellar initial mass function may not be significantly different in Galactic and embedded clusters. The differences in stellar densities at birth between the three ‘distinct’ modes may well be large enough to provide distinct initial mass functions implying that globular clusters may have significantly different initial mass functions than Galactic or embedded clusters, or a stellar population born in the distributed mode. Given these speculations, we remember however that Leinert et al. (1993) have found evidence that the mass ratio distribution of pre-main sequence binaries born in the distributed mode is best approximated with the Galactic field stellar mass function, which by its definition must be the result from the dominant mode of star formation. This finding casts doubt on the expected variation of the stellar mass function with star formation mode, but as yet the data are too poor to reject it.

In the light of our findings it is illustrative to turn our argumentation around. If observations of star forming regions affirm that the majority of stars form in embedded clusters that are dynamically equivalent to our dominant mode cluster then we have found an argument for the KTG(1.3) mass function which is independent of star count data and only rests on the correction of the mass-ratio distribution of G dwarf binaries for stimulated evolution in the dominant mode cluster.

### 6.3 The initial distribution of periods

Our initial period distribution was derived not by matching to the observational pre-main sequence data but by inverting the period depletion function (Fig. 6). The derived initial period distribution, which is valid for star formation in the dominant mode cluster ( $R_{0.5} \approx 0.8$  pc), agrees with the pre-main sequence observational constraints (Fig. 8) which have been obtained for distributed star formation (assuming incompleteness corrections do not significantly alter the observed distributions – see Section 1). This result suggests that the initial period distribution may be representative of star formation in general, and may not be a strong function of the star forming environment. This conclusion does not support the contrary assertion by Durison & Sterzik (1994), but future observations are needed to clarify this issue.

### 6.4 The dominant mode embedded cluster

The simulations of the dynamical evolution of our aggregates assumes virial equilibrium without any additional mass apart from that contributed by the stars. This is not a realistic model of embedded clusters which loose most (70–90 per cent) of the natal material through proto-stellar outflows and winds, as discussed in greater length by Mathieu (1986). Battinelli & Capuzzo-Dolcetta (1991) find that the majority of clusters have a lifetime of about 10 Myrs by which time most of the natal gas must have been removed rendering the cluster unbound. The orbital parameters of the young binary population will thus be ‘frozen’ after roughly  $10^7$  yrs.

Lada & Lada (1991) define a cluster as a group of stars having a mass density of at least  $1 M_\odot \text{ pc}^{-3}$ . Following this definition the aggregates considered here (Table 1) have a lifetime of about 250 Myrs (K3). Realistic models of embedded clusters must therefore include a background potential which evolves on a timescale of a few Myrs. Lada, Margulis & Dearborn (1984), Pinto (1987) and Verschueren & David (1989) consider the effects of a changing background potential on the stability of embedded clusters, but application to an initial aggregate of binary systems awaits to be done.

By adding a background potential but keeping  $R_{0.5}$  constant we increase the velocity dispersion in the cluster (assuming virial equilibrium). Since the cross section for collisions between binary systems is proportional to  $v^{-2}$ , where  $v$  is the relative velocity of two binaries well before the encounter (see equation 26 in Heggie & Aarseth 1992) we expect stimulated evolution to decrease. Thus we need to decrease  $R_{0.5}$  to increase the number density and thus the collision rates between the binary stars.

The destruction rate of binary systems is

$$\frac{dn_b}{dt} \propto -\frac{n_b^2}{v}, \quad (12)$$

(equation 26 in Heggie & Aarseth 1992), where  $n_b \propto (N_{\text{bin}}/R_{0.5}^3)$  is the number density of binary stars. Assuming a star formation efficiency  $\epsilon$  we have a total birth aggregate mass  $M_{\text{tot}}(0) = M_{\text{stars}}/\epsilon = N_{\text{bin}} \bar{m}/\epsilon$ , where  $\bar{m}$  is the mean binary system mass ( $0.64 M_\odot$ ). The velocity dispersion in a cluster is  $v \propto (M_{\text{tot}}/R_{0.5})^{1/2}$  so that

$$v_{\text{new}}^2 = \frac{1}{\epsilon} v_{\text{old}}^2, \quad (13)$$

where ‘new’ refers to the dominant mode embedded cluster, and ‘old’ refers to the dominant mode cluster.

Assuming that the ‘dominant mode embedded cluster’ has a lifetime  $\eta$  times smaller than our ‘dominant mode cluster’ [ $(N_{\text{bin}}, R_{0.5}) = (200, 0.8 \text{ pc})$ ] we need to ‘speed up’ the destruction of binary systems by a factor of  $\eta$ , i.e.  $dn_{b,\text{new}}/dt = \eta dn_{b,\text{old}}/dt$ . We obtain  $n_{b,\text{new}}^2 = (\eta/\epsilon^{1/2}) n_{b,\text{old}}^2$ . Thus

$$\frac{R_{0.5,\text{new}}}{R_{0.5,\text{old}}} = \frac{\epsilon^{1/2}}{\eta^{1/6}} \left( \frac{N_{\text{bin},\text{new}}}{N_{\text{bin},\text{old}}} \right)^{1/3}. \quad (14)$$

If  $N_{\text{bin},\text{new}} = N_{\text{bin},\text{old}} = 200$  binaries form in an aggregate and  $R_{0.5,\text{old}} = 0.8 \text{ pc}$  (the dominant mode cluster) then we obtain Table 3 for  $R_{0.5,\text{new}}$ :

**Table 3.** Expected half mass radii  $R_{0.5,\text{new}}$  of the dominant mode embedded cluster

$\eta$	5	10	20	30
$\epsilon$				
0.1	0.505 pc	0.450 pc	0.401 pc	0.375 pc
0.2	0.534	0.477	0.425	0.397
0.3	0.553	0.493	0.439	0.411
0.4	0.567	0.505	0.450	0.421

Thus  $0.37 \text{ pc} < R_{0.5,\text{new}} < 0.57 \text{ pc}$  if we demand *dynamical equivalence* with our dominant mode cluster! These  $R_{0.5,\text{new}}$  are ‘initial’ values, i.e. *at the time of star–gas decoupling* (see also Section 6.6), and the observed embedded clusters are likely to have already expanded from their initial configuration. If initially  $R_{0.5} = 0.37 \text{ pc}$  then the approximate average inter-binary spacing (approximately 0.1 pc) corresponds to a period of  $10^{9.14}$  days for a binary system of total mass  $0.64 M_\odot$ . This is larger than the maximum period in our initial period distribution, and so *we do not expect erosion of the period distribution alone from crowding* in the dominant mode embedded cluster.

The results of our simulations which neglect the gas component thus appear to be robust because  $R_{0.5,\text{new}}$  does not change significantly even if the effect of the (initially mass dominating) gas component is taken into account.  $R_{0.5,\text{new}}$  is not significantly different to  $R_{0.5,\text{old}} = 0.8 \text{ pc}$ . Numerical simulations will, however, be necessary to verify these conclusions.

## 6.5 Small groups of stars as the dominant mode of star formation?

In Section 6.2 we have discussed some alternatives to our model (e.g.: a different mass function and distributed star formation being the most common mode) which we find are not consistent with the observational

data. In this context we discuss the possibility that binaries may form preferentially in small groups. McDonald & Clarke (1993) assume stars form in small groups of three to ten and that the most massive stars in these groups form binary systems by dynamical capture. They obtain a mass ratio distribution as observed by Duquennoy & Mayor (1991) despite using a mass function similar to that adopted here (equation 1), as well as a decreasing proportion of binary systems with decreasing primary mass. The overall proportion of binaries thus formed is, however, much smaller than  $f_{\text{tot}}^{\text{obs}} \approx 0.5$ .

Energy dissipation in the remaining circum-protostellar material is likely to be very important in raising the proportion of binaries formed by capture. McDonald & Clarke (1995) include a simple model of energy dissipation in circumstellar discs. They obtain a larger proportion of binaries than in the absence of discs (McDonald & Clarke 1993). The proportion of G- and M-dwarf binaries of the resulting stellar system population is about correct if the preferred initial group size is  $N \approx 10$  (their fig. 13). If stars are distributed randomly throughout the group then the component masses in binary systems are uncorrelated (the probability of two closest neighbours dissipating enough relative kinetic energy to form a bound binary is increased). This is shown in their fig. 11 but is not consistent with the main sequence G dwarf mass ratio distribution (Kroupa & Tout 1992). This result would appear to indicate that dissipative capture in small groups ( $N \approx 10$ ) of proto stars may not be able to account for the distribution of dynamical properties of stellar systems in the Galactic disk.

In Section 6.6 we speculate that typical star formation in the Galactic disk may, however, be characterised by the formation of aggregates containing tens of dissipative small- $N$  groups.

## 6.6 The star formation process

For star formation (i.e. for times *during* the first  $10^5 - 10^6$  yrs) we may interpret our results as follows:

Our assumptions and results would appear to be valid if most proto-stellar binary systems would form by dissipative capture in small groups of about ten accreting proto-stars that are clustered in aggregates of tens of such groups (Section 6.5). Aarseth & Hills (1972) show that subclustering enhances the proportion of binary systems formed by capture even if stars are approximated as point particles. Murray & Clarke (1993, in their section 5) similarly find a higher production of binary systems by dissipative capture in clumpy initial conditions, the virial velocities in each subgroup being sufficiently small to imply capture rather than disc destruction. This may produce uncorrelated component masses (see Section 6.5), a dynamically relaxed eccentricity distribution and a majority of proto-binaries with large periods. If the evolution timescale of the individual groups is sufficiently smaller than the evolution timescale of the whole aggregate (as is the case in the simulations of Aarseth & Hills 1972) then we would arrive at our results. For example, a group with  $R_{0.5} = 0.04$  pc consisting of 10 stars has a relaxation time (Spitzer & Hart 1971) of about  $T_{\text{relax}} = 0.07$  Myrs, whereas an aggregate with  $R_{0.5} = 0.8$  pc consisting of 200 binaries has  $T_{\text{relax}} = 10$  Myrs.

Groups of about 10 accreting protostars may result from fragmentation during protostellar collapse and fragmentation of accretion disks. We need magnetohydrodynamics to study the very early evolution, and in fact the onset of star formation itself (see e.g. Patel & Pudritz 1994). Modern computational facilities, and in part our knowledge of the relevant physics (e.g. opacities, radiation transfer etc.) do not presently allow this time scale to be studied in great detail. However, despite the present limitations the Cardiff group has been obtaining interesting numerical results on the formation of binary systems and stellar aggregates in shocked and colliding molecular cloud clumps (see e.g. Turner et al. 1994). It remains unclear whether the proto-stellar clumps remain in virial equilibrium in their cloud core, or whether they freeze out of the core with smaller velocities than the characteristic velocity dispersion associated with the turbulent molecular gas, whether a violent cold collapse occurs, or whether the proto-stars sink to the centre of the potential well with dynamical friction (Just, Kegel & Deiss 1986) reducing their rate of infall, and how the gaseous medium reacts, i.e. how much energy is deposited from the motion of the proto-stars into the molecular gas, which in turn affects the star-gas coupling and gas removal.

The protostars will decouple from the gas after the first  $10^5 - 10^6$  yrs have passed. This occurs because the gas density remaining within the young aggregate decreases owing to gas accretion onto the stars and gas removal by very young active pre-main sequence stars.

The results of our simulations uncover the most common dynamical structures which may be established *after* this early time.

### 6.7 Hierarchical star formation?

Our speculation in Section 6.6 suggests that the notion that aggregates of groups of about 10 proto stars may condense out of a molecular cloud core may be consistent with our assumptions (Section 3.2.1).

It is now known that in the Taurus–Auriga star forming regions pre-main sequence stars are distributed spatially in a fractal or self-similar way (see e.g. Larson 1995). The stars are clustered hierarchically, with smaller groupings within larger ones over a considerable range of scales. This self-similar clustering breaks down on scales smaller than 0.04 pc, most if not all stars being members of binary systems. Taurus–Auriga is a low-density star forming environment, i.e. the stars have formed in the distributed or isolated mode.

Our inverse dynamical population synthesis suggests that most stars in the Galactic disk may have formed in embedded clusters rather than in the distributed mode. The properties of the binary star population in embedded clusters are unknown. Inverse dynamical population synthesis suggests that after star–gas decoupling most stars may be in about 1 pc large aggregates of a few hundred binary systems.

Binary systems would thus appear to be the primary subunit of star formation in *both* the isolated *and* clustered star formation mode.

It is thus possible that star formation produces hierarchical clustering which evolves, after a few dynamical timescales of the various fractal subunits, to the structures that about 1–10 Myr old populations are observed to have. Low-density star forming environments may evolve significantly only on the smallest scales thereby producing a large proportion of binaries. The high-density star forming regions (the ‘clustered mode’) would contract to embedded clusters. If this is correct then the initially hierarchical structure would first produce predominantly binary systems with uncorrelated component masses, a dynamically relaxed eccentricity distribution and predominantly long periods. These binary systems would then interact in the evolving aggregate to produce the Galactic disk stellar population after the aggregate has disintegrated after approximately  $10^7$  yrs. In even higher density environments with relatively high star formation efficiency the initial hierarchical distribution may evolve to Galactic clusters.

## 7 CONCLUSIONS

Recent observational data (Figs. 1 and 2) suggest that the distribution of orbital periods of pre-main-sequence binary systems differ from main sequence orbital distributions such that star-forming regions have a significantly higher binary proportion.

We assume that all stars form in aggregates of 200 binary systems (a reasonable first assumption, see Section 3.2.2) with component masses paired at random from the KTG(1.3) mass function derived from a thorough analysis of local and deep star count data by Kroupa et al. (1993). This mass function contradicts the observed mass function of secondaries in binaries with G dwarf primaries which has a deficit of low mass companions (Kroupa & Tout 1992), but it is consistent with the distribution of brightness ratios for pre-main sequence binaries (Leinert et al. 1993). These assumptions will be verified by observations if our results are a reasonable approximation of the star-formation process.

Assuming an initially flat  $\log_{10}P$  distribution ( $P$  in days,  $3 \leq \log_{10}P \leq 7.5$ ), which is consistent with the observed pre-main sequence period distribution, we find that a cluster with half mass radius  $0.25 \text{ pc} < R_{0.5} < 0.8 \text{ pc}$  has the correct dynamics to lead to a proportion of main sequence binary systems consistent with the observed value ( $47 \pm 5$  per cent, Fig. 3). A cluster with  $R_{0.5} = 0.8 \text{ pc}$  leads to a mass-ratio distribution for binaries with G-dwarf primaries as observed (Fig. 4), and to a depletion of the period distribution with increasing period which is similar to the observed main sequence distribution (Fig. 5).

This finding suggests that the dominant mode of star formation in the Galactic disk may be clustered star formation. We refer to our  $R_{0.5} \approx 0.8 \text{ pc}$  cluster as the *dominant mode cluster*. We estimate that the initially mass dominating gas component in embedded clusters does not significantly affect this result (Section 6.4). Presently available observations of star forming regions appear to confirm our result (Section 6.1), but the sample of star-forming regions has to be extended before this result can be taken as conclusive.

Hardening and softening of binary systems in the stellar aggregates considered here (Table 1) does not significantly change the numbers of orbits with  $\log_{10}P < 3$  and  $\log_{10}P > 7.5$ , respectively (Section 4). The

orbital depletion function can be inverted to derive an initial distribution of periods (Section 5). We term the evolution of orbital elements due to mutual perturbation of neighbouring systems *stimulated evolution*.

The observational data of pre-main sequence binaries are valid only for distributed or isolated star formation. We find that the initial period distribution (equation 11) is very similar to the observed period distribution of pre-main sequence binaries. This initial period distribution was derived for clustered star formation. Significant variation of binary star properties with star formation environment is thus not suggested by the results obtained here but future investigation is required to confirm this assertion. If it is found that the observed orbital distributions in different star forming regions differ then we caution against inferring that the primordial binary star properties depend on local conditions (Durison & Sterzik 1994) unless it can be established that the observed difference cannot be due to stimulated evolution. In comparing stellar dynamical properties (mass function, orbital parameters) in different star-forming regions it is important to establish the stage of dynamical evolution of these regions. For example, in K3 we argue that a distributed population of young stars in the L1641 molecular cloud may have originated in by now dissolved aggregates. If this is true then this population will have a binary proportion of 50–60 per cent and a period distribution similar to the main sequence distribution. These properties (if verified) would be different to the Taurus–Auriga pre-main sequence binaries.

We find evidence that the mass function power law index  $\alpha_1$  (equation 1) may depend on the stellar number density at birth (Section 6.2). Our assumptions and results appear to be consistent with hierarchically distributed star formation (Section 6.7).

We conclude with the following question pertaining to the origin of stars in the Galactic disk: Once star formation begins, have most of the molecular cloud cores evolved in parameter space (pressure, temperature, turbulent kinetic energy etc.) in such a way that the subsequent dynamics of the aggregate of stars that freeze out of the core is *equivalent* to that of our dominant mode cluster [ $(N_{\text{bin}}, R_{0.5}) \approx (200, 0.8 \text{ pc})$ ] (Section 4.3)? It would appear to be of great interest to conduct additional simulations with the period, eccentricity and mass ratio distributions of binaries at birth derived here but other virial ratio (e.g. violent relaxation), changing background potential leading to shorter time scales for stimulated evolution, and other mass and/or size of the initial aggregate of binaries, to seek those initial configurations which give the same final distributions of stellar dynamical properties as are observed for Galactic field stars and which are obtained with our dominant mode cluster. An initial non-negligible proportion of triple and quadruple systems will be required to explain the number of such systems in the Galactic disk and in star forming regions (K2).

This may be extended to include non-dominant modes at the present epoch of star formation, such as rich Galactic clusters or globular clusters, to investigate if our initial stellar dynamical properties are consistent with the observed stellar dynamical properties in such clusters.

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### Figure captions

**Figure 1.** Collation of all currently available data on the period distribution of late-type binary systems (Section 2). Solid dots show the distribution of orbits for solar-mass main sequence binary systems (Duquennoy & Mayor 1991), open circles indicate the preliminary distribution of periods for K-dwarf main sequence binary systems (Mayor et al. 1992) and stars represent M dwarf binaries (Fischer & Marcy 1992). The area under each distribution is the proportion of orbits among all G, K and M stellar systems. The distribution of orbital periods of low-mass pre-main sequence stars, obtained by transforming from the projected separations, are shown by open triangles (Simon 1992), open squares (Leinert et al. 1993 and Richichi et al. 1994), open star (Reipurth & Zinnecker (1993) (on top of an open square at  $\log_{10}P = 6.5$ ) and cross (Ghez et al. 1993). The proportion of pre-main sequence binaries with  $\log_{10}P < 2$  is indistinguishable from the Galactic field distribution (Mathieu 1992, 1994).

**Figure 2. Top panel:** the distribution of mass ratios for main sequence solar-type systems of all periods (Section 2). The filled dots show the distribution derived by Duquennoy & Mayor (1991). The open circles denote their mass ratio distribution corrected here for the bias in short-period binaries (Mazeh & Goldberg 1992) by adding the corrected short- and long-period distributions shown in the middle panel. The upper dotted, middle dashed and lower dotted curves represent the KTG( $\alpha_1$ ) mass function with  $\alpha_1 = 1.85, 1.3, 0.7$  (equation 1), respectively, which is the 95 per cent confidence interval for  $\alpha_1$  (Kroupa et al. 1993). The units of the ordinate are the number of systems per bin. **Middle panel:** the corrected distribution for short period ( $\log_{10}P < 3.5$ ) binaries constructed from the data given by Mazeh et al. (1992) is depicted by open circles. The one-sigma error range is indicated by the dotted lines. The distribution of orbits for long periods ( $\log_{10}P > 3.5$ ) is shown by the filled dots and was obtained from the data presented by Duquennoy & Mayor (1991). Both distributions are normalised to unit area. **Bottom panel:** the short period ( $\log_{10}P < 3$ , open circles) and long period ( $\log_{10}P > 3$ , solid circles) eccentricity distributions from Duquennoy & Mayor (1991), both normalised to unit area.

**Figure 3.** Evolution of the total binary fraction with time for the first four binary star aggregates in Table 1 (Section 4.2). Binary stars also form in the aggregates by capture, and the proportion of such binaries is shown for the single star aggregates (Table 1 with  $N_{\text{bin}} = 0$ ).

**Figure 4.** The initial (dotted histogram) and final (solid histogram) distribution of secondary masses for primaries with a mass in the range  $0.85 M_{\odot} < m_1 < 1.1 M_{\odot}$  for the first four binary star aggregates in Table 1 (Section 4.2). The observational data in the top panel of Fig. 2 have been scaled to the initial models at  $m_2 > 0.6 M_{\odot}$ .

**Figure 5.** The initial (dotted histogram) and final (solid histogram) distribution of periods for the first four binary star aggregates in Table 1 (Section 4.2). The observational data are as in Fig. 1.

**Figure 6.** The depletion function  $c_{P,i}(1 \text{ Gyr})$  (equation 8) introduced in Section 5.1 for the first four aggregates (Table 1): open stars ( $R_{0.5} = 2.53 \text{ pc}$ ), solid circles ( $R_{0.5} = 0.77 \text{ pc}$ ), open circles ( $R_{0.5} = 0.25 \text{ pc}$ ) and crosses ( $R_{0.5} = 0.08 \text{ pc}$ ).  $c_{P,i} > 1$  implies a gain in the  $i$ th  $\log_{10}P$  bin.

**Figure 7.** The corrected initial period distributions (equation 9a) obtained in Section 5.1 by applying the depletion function of Fig. 6 to the main sequence data of Duquennoy & Mayor (1991) shown here as solid dots. Results for each of the first four aggregates (Table 1) are shown: dotted line ( $R_{0.5} = 0.08 \text{ pc}$ ), long dashed line ( $R_{0.5} = 0.25 \text{ pc}$ ), solid line with large solid dots ( $R_{0.5} = 0.77 \text{ pc}$ ) and dot-dashed line ( $R_{0.5} = 2.53 \text{ pc}$ ). The thick solid line is the initial period distribution adopted as the first iteration (equation 10).

**Figure 8.** a) First and b) second iterations towards the initial period distribution. In both panels the continuous initial distributions (equations 10a and 11a) are shown as the thick solid line. The distribution of orbits picked initially from these distributions are shown by the dotted histograms, and the final period distributions are shown by the solid histogram. The observational data are as in Fig. 1.